

# Structural transition of force chains observed by mechanical spectroscopy

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The dissipation properties of a fine sand system are investigated by a low-frequency mechanical spectroscopy. The experiments show many interesting profiles of the relative energy dissipation, which imply that some structural transition of force chains in dense granular media has occurred. The following data and discussion indicate that the transition of force chains will lead to the small deformation of arrangement in the granular system, which is responsible for the historical effects. We hope this research can improve our knowledge of the microstructure of the granular materials.

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## I. INTRODUCTION

Granular materials [1] are ubiquitous in everyday life, but a satisfying comprehension of their complex dynamics has not yet been achieved. Force chains [2, 3] play a key important role in understanding the static structure and dynamics properties of the densely packed granular materials, such as jamming [4], sound transmission [5], force propagation [6] and memory effects [7]. Many experiments and theoretical simulations have been carried out and focused mainly on the characters of force chains [8, 9]. The most common method used to examine force chains and their key characteristics is to visualize these contact forces by stress-induced birefringence within assemblies of photoelastic grains [2, 3]. Experiments and simulations [4, 10] all show that the transmission of force chains appears as a complex force network that is highly ramified and distributes inhomogeneous throughout the whole system. In fact, the force chains are quite sensitive to small perturbations in the packing geometry of the grains [2, 5, 6, 11]. For example, if a granular material is driven by a small shear, the force chains will change dramatically and evolve in a complex random way [6, 12, 13]. So recently an alternative method has been developed to probe the force chains and its change [14, 15]. This method is based on the relationship between the energy dissipation and the structural changes of force chains in the granular system, which is related to the stress relaxation process [11].

In Ref. [14, 15], the dissipation properties of a sheared granular medium have been studied and a simple rheological model is presented, which suggests that small slides in the inhomogeneous granular materials are responsible for the energy dissipation. However, the microscopic picture about the slides is unclear. In this work, our investigation will concentrate on the details of the small slides in the microscopic picture, which we call the structural transition of force chains. The mechanical spectroscopy exhibits a fact that a small change of the force network

structure in granular system can lead to a corresponding energy dissipation. In addition, our experimental data show that in the granular system the arrangement and the volume fraction of the grains will change with the structural transition of force chains. The following discussion indicates that it is the small change of the arrangement that leads to the well-known historical effects, which mean that the state and properties of granular materials are quite dependent on their past history.

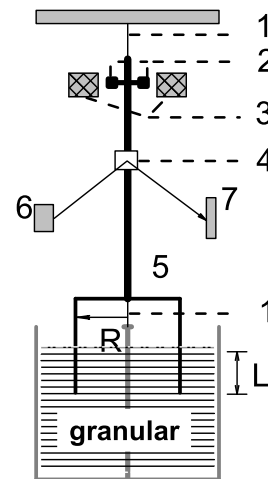


FIG. 1: Sketch of the forced torsion pendulum immersed into the granular medium. 1, suspension wires; 2, permanent magnet; 3, external coils; 4, mirror; 5, cylinder (made of aluminum alloy with inner radii  $R$ ) un- or covered by a fixed layer granules; 6, optic source; 7, optic detector

## II. EXPERIMENT

In the experiment, the mechanical spectroscopy measurements were conducted on a developed low-frequency inverted torsion pendulum using the forced-vibration method. Fig. 1 shows the sketch of the pendulum. The

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inverted torsion pendulum consists of a cylinder being able to rotate around its axis, but prevented from moving sideways by two suspension wires fixed to two ends of the cylinder. The cylinder is forced into torsional vibration by a time-dependent force  $F(t) = F_0 \sin(2\pi\nu t)$ , exerted by applying a pair of permanent magnets fixed to the pendulum and external coils (circulating an ac current), where  $\nu$  is the forced frequency. The angular displacement function of the cylinder,  $A(t)$ , is measured optically. In the case here, the response of the argument  $A(t) = A_0 \sin(2\pi\nu t + \delta\alpha)$ , where  $\delta\alpha$  is the phasic difference between  $A(t)$  and  $F(t)$ . According to this measurement technique, the relative energy dissipation (RED) is calculated by measuring the loss angle between applied stress and resulting strain. Meanwhile, the relative modulus (RM) is calculated from the ratio between the stress and strain.

Before each experiment, the granular system is flattened and vibrated by external vibrations to ensure the accuracy of the measurements. And the whole system is placed on an antivibrational table to prevent undesired vibration-induced effects. In our experiments the granular materials are composed of fine sand sieved diameter  $d = 0.054 \sim 0.11\text{mm}$  and the maximum angular displacement is below  $0.4^\circ$  ( $0.4\pi R/180 < 0.1d$ , which indicates that the real displacement of the cylinder is less than  $0.1d$ , relative to the averaged particle size. So the system can be considered as quasistatic.). The forced frequency  $\nu = 0.6\text{Hz}$  or  $\nu = 1.0\text{Hz}$  is chosen, which is well below the inherent frequency of the pendulum (about  $36\text{Hz}$ ).

### III. RESULTS

In our experiments the RED and the RM are given by  $\eta = \tan(\delta\alpha)$  and  $G_0 = F_0/A_0$ , respectively. The mechanical spectroscopy (RED and the RM) as a function of the amplitude  $A_0$ , are shown as follow.

Figure 2 shows clearly the relative energy dissipation increases with the amplitude. Meanwhile, we observe that the relative modulus decreases monotonously with increasing amplitude. These data are obtained when we swept the amplitude down repeatedly. Whether there is hysteresis in the system, i.e. are the peaks still there if the forced amplitude is swept up and down? This is an important question and relates to the historical effects have been found in weakly vibrated granular systems [18]. As shown in Fig. 3(a), when we increase the amplitude step by step, we obtain a slowly decreasing RED. In succession, when the amplitude is swept down slowly, two different profiles (RED and RM) are obtained, as the profile (2) in Fig. 3 shows. We find that the differences between them are obvious: a rather small RED is obtained and there is a loss peak, while RM increases monotonously with decreasing amplitude. This is the historical effects, which indicate that the state and properties of granular materials are quite dependent on their past history.

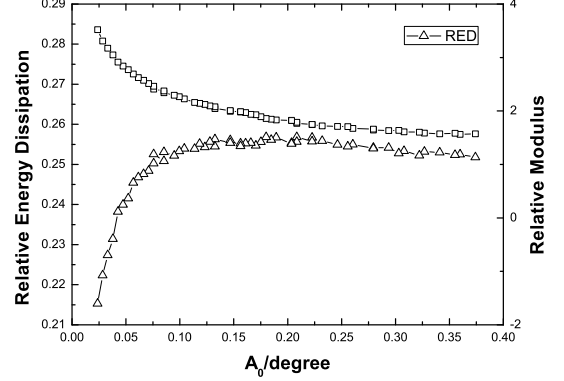


FIG. 2: The RED ( $\triangle$ ) and RM ( $\square$ ) versus the amplitude  $A_0$ (degree) in a fine sand system with the oscillating cylinder covered by a layer of grains, for the frequency  $\nu = 0.6\text{Hz}$  and the immersed depth  $h = 60d$ .

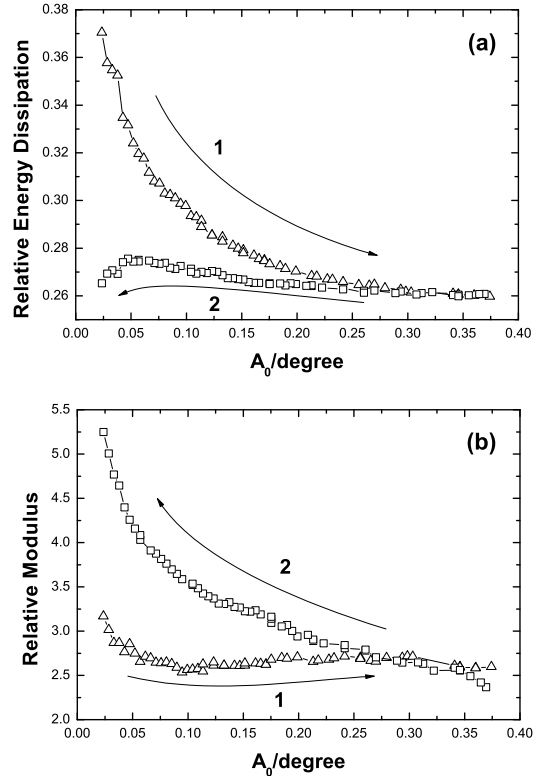


FIG. 3: The historical effects of granular materials. (a) The RED versus the amplitude with different path. (b) RM versus the amplitude with different path. The immersed depth  $h = 60d$  and the forced frequency  $\nu = 0.6\text{Hz}$ .

#### IV. STRUCTURAL TRANSITION OF FORCE CHAINS

In Ref. [14, 15], a simple rheological model is presented to reproduce the RED measured in granular materials. In the model, the granular medium is characterized by slide unit and spring unit. The model suggests that small slides in the inhomogeneous granular materials are responsible for the energy dissipation. However, the microscopic picture about the slides is unclear. Below we will try to give some details of the small slides in the microscopic picture and then give a explanation of the historical effects.

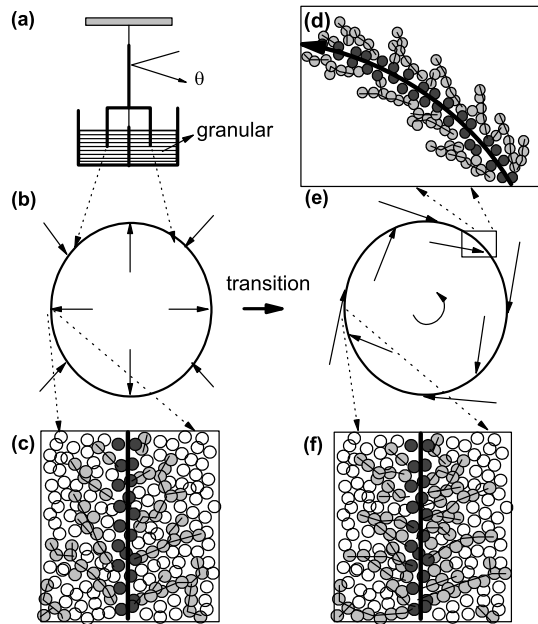


FIG. 4: The structural transition of force chains. (a) Sketch of the forced torsion pendulum immersed into the granular medium. The cylinder is covered by a layer sand glued on with epoxy. (b) Schematic (horizontal direction) and (c) the microscopic picture (vertical direction) of the distribution of force chains in a steady state. (e) Schematic (horizontal direction) and (f) the microscopic picture (vertical direction) of the distribution of force chains opposing the rotation of the pendulum. The dark grains are glued on the cylinder by an epoxy. The force chains are represented as bonds connecting the light grey grains. Picture (d) shows the force chains in a horizontal layer near the cylinder. In this picture, the grains besides those on the force chains have not been shown.

Granular materials are relatively discrete medium and force transmission through a granular system can only occur via the interparticle contacts. Under the gravity force, the force distribution in a static packing of grains in a cylinder should be inhomogeneous and could form many force chains [7, 10]. Near the cylinder, the spatial force distribution will be organized along directions

almost normal to the cylindrical probe [14], where maximum strain be built up between chains of grains, due to the gravity. These grain chains of forces have two characters: first, the contact force between them is rather stronger and carries almost all the weight load of the above grains, and second, a chain is a quasilinear arrangement of three or more grains [9]. The chain lengths are defined as the grain number in the chain, e.g., the shortest chain is two-grain chain. Meanwhile the magnitude of the contact force denotes the correlation strength of the chain. Here we present a sketch of the grain chains in Fig. 4(b) and (c). Fig.4(c) shows a part of the microscopic picture (vertical direction) of the distribution of force chains in static state. As this picture shows, these force chains are oriented almost in the vertical direction because of the weight of the grains. In other words, the forces originate from the weight of the grains and the weight of an above particle is transmitted to a neighbor underlying particle, as shown in q-model [2, 8]. So we often find the force chains arrange as roots of trees. In addition, we know that the qualities and strengths of chains will increase with the depth (“Janssen effects”).

In the granular systems, the grains can not move independently. More often some degree of freedom of a grain are partially frozen, so that the motion of the grains is a correlated motion. When a shear stress is applied to the granular material, rather than deforming uniformly, the system such as dry sand develop shear bands [16] — narrow zones of strongly correlated particles, with essentially rigid adjacent regions. Similarly, when a cylindrical probe is rotating in the granular medium, the spatial force distribution around the probe will be organized along directions almost tangent to the cylinder, where the maximum stress also build up many chains of grains. These correlations of the grain chains have been shown by the radial profiles of azimuthal velocity in Ref. [12, 17]. Here we present a sketch of the grain chains under shear stress in Fig. 4 (e) and (f). Compared the conditions before the shear, many differences will appear as follow. First, under shear stress more neighbor grains of the cylindrical probe will join in the force chains, although, the configuration of grains could not change, as Fig. 4 (c) and (f) show. Second, the directions of force chains will change a lot, i.e., the force chains oriented almost in the horizontal direction in contrast to almost in the vertical direction. The reason is that the shear stress changes the origin of the force chains. Then the shear stress leads to the structural changes of the force chains. Except for changing the arrangement of the force chains, the applied shear will also influence the characters of the force chains, e.g., the chain lengths will get longer and the correlation strength of the chains will become stronger. In order to clearly show the schematic of force chains, we give a microscopic picture of the force chains in a horizontal layer near the cylinder as shown in Fig. 4 (d), where the other grains besides the grains on the force chains have not been shown.

Here we call this change of the force chains with-

out configurational change as the structural transition of force chains. Now let us discuss the relationship between the structural transition of force chains and the energy dissipation. The definition of force chains indicates that the drag force resisting a solid object moving slowly through a granular medium originates not only in the grains immediately in front of the object but also in the successive layers of grains supporting them. When a cylindrical probe begin to rotate in the granular medium, the spatial force distribution around the probe will change dramatically, as shown above. During this structural transition of force chains, some change of configuration in microscopic length scale, such as the formation and break of adhesive junctions between the surface asperities, and other forms of localized dissipative processes, must have occurred [14]. Figure 2 shows clearly the relative energy dissipation in these processes. When we increase the vibration amplitude, the structural transition, such as the break of adhesive junctions between the surface asperities, will increase, i.e., the transition quantity will increase. When the transition quantity increases the energy dissipation must will increase. As expected, the experimental results also show that the energy dissipation increases with the amplitude. Meanwhile, we observe that the relative modulus decreases monotonously with increasing amplitude, which indicates the soften of the dense granular system under shear strain [15]. These analysis indicate that the structural transition of force chains leads to the energy dissipation and the dissipation will increase with the transition quantity.

In our experiments, the granular materials are composed of fine sand sieved diameter  $d = 0.054 \sim 0.11\text{mm}$ . In this scale, granular matter is a well-known example of athermal system, that is a system where classical thermodynamics does not apply since thermal energy ( $k_B T$ ) is insignificant compared to the gravitational energy of a macroscopic grain. A static packing of grains is therefore in a metastable state, indefinitely trapped in a local minimum of the total potential energy. However, the forced vibration will break the jamming of granular packing [13]. From Ref. [19] we know that after a succession of high amplitude vibration a rather looser packing of grains (the volume fraction is small) is obtained. However, we do not know what about the mechanical spectroscopy in such a looser packing of grains under low-amplitude shear. And we do not know how the volume fraction of grains influences the energy dissipation of granular system. In the following we will discuss the energy dissipation in the condition of historical effects. As we know, the data in Fig. 2 are obtained in the experiments performed in the direction that the forced amplitude is swept down repeatedly. In the process we decrease the amplitude step by step and we obtain a slowly decreasing RED. The beginning of the profile (1) in Fig. 3 shows the dissipation at low amplitude obtained immediately after a succession of high amplitude vibration. In other words, at this moment the granular packing is rather looser. But at the beginning a rather larger RED and a rather lower

RM are obtained. Then Fig. 3(a) shows that the RED decreases monotonously with increasing amplitude. In succession, when the amplitude is swept down slowly, a denser packing is obtained [19] and two different profiles (RED and RM) are obtained, as the profile (2) in Fig. 3 shows that a rather small RED is obtained, while RM increases monotonously with decreasing amplitude (see Fig. 3(b)). The above discussion implies that the energy dissipation is related to the volume fraction of granular system, i.e., the looser packing is more dissipative. Considering that the energy dissipation of granular system increases with the transition quantity of force chains, we can say that the structural transition will occur more easily in the looser packing.

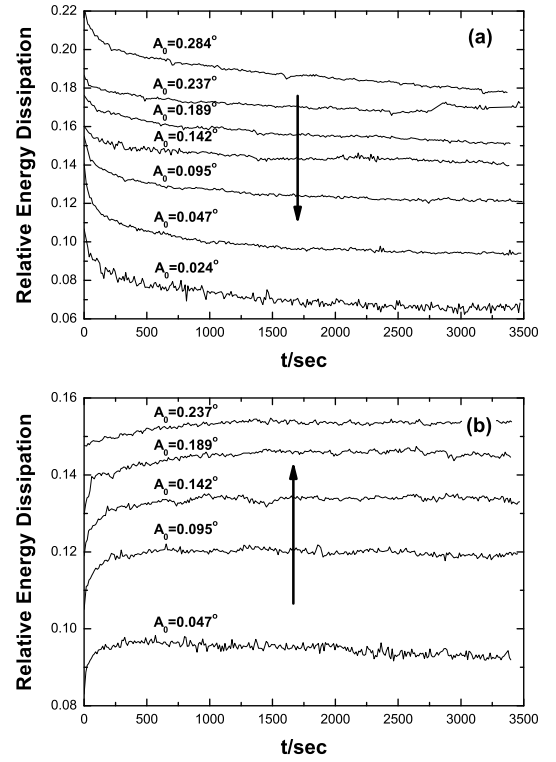


FIG. 5: The RED versus the time with different amplitude  $A_0$ , as noted. Firstly the amplitude is swept down (a), and then the amplitude is swept up (b). The immersed depth  $h = 70d$  and the forced frequency  $\nu = 1.0Hz$ .

In order to understand the historical effects better, below we will discuss the aging effects in granular materials. Fig. 5 shows the RED as a function of the time with different amplitude  $A_0$ , as noted. At the beginning of all these experiments, the packing is rather looser. Then under a series of vibrations with the amplitude  $A_0 = 0.284^\circ$ , the granular system will be compacted [19]. According to the relationship between the volume fraction and the energy dissipation of granular system, we know that when the granular system is compacted the RED will decrease

with the vibration time. As expected, Fig. 5(a) shows the results: firstly the RED decreases quickly and then slowly approaches a saturation value. In other words, at the beginning the structural transition of force chains will occur continuously with the energy dissipation of system. Then the transition quantity will decrease with the compaction of the granular packing, which is the reason of the decrease of energy dissipation. These analysis indicate that the structural transition of force chains has slowly changed the arrangement of the granular materials, i.e., the compaction of granular materials. And it is this arrangement change that leads to the change of energy dissipation with vibration time. The profiles in Fig. 5(a) can be well fitted with an exponential decay law, which is a fundamental character of the relaxation of granular systems [19]. This is the aging effects. Fig. 5(a) presents many similar profiles with different amplitude. The difference between them is that the quantity of RED decreases with decreasing forced amplitude. However, when we increase the amplitude step by step, we obtain some different profiles. As Fig. 5(b) shows, at the beginning the RED increases quickly and then slowly approaches a saturation quantity, what indicates that the relaxation of granular system has occurred and the arrangement of granular system must have changed. As mentioned above, it also is the structural transition of the force chains that leads to this change. However, the profiles in Fig. 5(b) follow the exponential grow law, which shows that the trend of the profiles is opposite with the profiles in Fig. 5(a). These differences between the profiles indicate that the arrangement of the grain packing changes differently when we increase the amplitude step by step. This is a process that the granular packing changes looser, in contrast to the compaction. In addition, the results confirms an earlier experimental observation: as the intensity of vibration decreases both the volume fraction of stationary and the compaction time increases [19]. The reason is that the transition quantity will be very small when the amplitude of vibration is lower. In according to the above analysis, we know that the forced vibration actually changes the microstructure (the volume fraction and the arrangement) of the granular system because of the slowly structural transition of force chains. This change of the microstructure is associated with the distribution of the various-size empty space between grains, i.e., the defects motion and annihilation, which is the fundament of the historical effects.

The above discussion indicates that the transition of force chains is a relaxation process accompanied with the energy dissipation, as shown in the explanation of experimental data. The mechanical spectroscopy also show that a minute change of the arrangement of the granular system is sufficient to significantly change the amplitude response. It is these changes of the arrangement that are responsible for the well-known historical effects in the granular system preparation.

## V. CONCLUSION

No body will be surprised that when he moves a solid intruder in a granular medium some deformations and some shear bands near interface occur [1, 16]. And we all know that the shear bands are the key to explain the “flowability” of granular materials. However, the shear banding behavior is very complex and is difficult to study. Many experiments and theoretical simulations have carried out and shown that the shear band thickness and shape are dependent on the shear strain and the boundary roughness conditions [6, 20]. Here we investigated the structural transition of force chains in the shear bands from a view of energy dissipation. We found that the shape of the shear bands will change with the shear stress, as the microstructure of force chains shows. Of course, there are also many questions in understanding what happens on the grain scale [21].

Different from the most common method to study force chains using birefringent materials [2, 3], our investigation focus on the relationship between the energy dissipation and the force chains in granular systems, which gives a microscopic picture of the force chains in dense granular materials. In the experiments, we observed many interesting physical effects, such as the historical effects and aging effects. The results indicate that some structural transition of force chains, even changes of arrangement, have occurred. While the energy dissipation in this regime is a dynamic quality, we find that it is determined by the static structure of the medium. So we can say this experiment offers a new bridge between recent developments in understanding static configuration in granular media and the dynamic properties.

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